Robust Control Frequency Analysis of a Moving Walking Bipedal Robot

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***Abstract*- Dynamic equations of the biped robots are easily obtained by the Lagrangian method. Because of terrible nonlinearity of these dynamic equations, the actuator's dynamic is employed to linearize these equations. By this way, the heavy nonlinear equations are replaced with a second order linear system which their coefficients are achievable by the various identification methods. Here Least Square method is utilized to determine these second order transfer function's coefficients. Employing a set of linear transfer function for each joint and also considering unstructured uncertainty, an *H*∞ controller is designed and the Robust Stability and Robust Performance criteria are to be handeled with the μ analysis theory.**

I. INTRODUCTION

Design of a biped Robot is a complicated work. They usually have many degrees of freedom. Biped robots have a nonlinear kinematics and dynamics, so they are not easily modeled. Biped Robot's parameters may also be unknown or vary with the environment's condition [4,5]. The environment is usually unknown and the surface of motion can be different as declined, stair and so on. Therefore, biped robots need to be controlled [6,7]. Too many studies and investigations are done in the field of biped robots and also various control systems are designed for them[8]. Here, a robust *H*∞ controller is designed for a 7 DOF biped robot moving on a declined surface.

II. DYNAMIC MODELING OF THE SYSTEM

Using dynamic equations a control system can be designed. Dynamic equations of this robot are easily obtained by the Lagrangian method and can be shown as (1):

$$M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) = \tau \tag{1}$$

Which symmetric matrix M is the inertia matrix, V is a vector containing centrifugal and corioles components and G is gravitational vector.

III. LINEARIZATION OF DYNAMIC MODEL

Dynamic equations of biped robots are extremely nonlinear and need to be linearized for designing *H*∞ controller. One way for linearization of these equations is differentiating around a special point, but in the field of biped robots such special point is not defined. Another approach for linearization is dividing the path of the robot into several points and then differentiating around them. In this case, a set of linear time varying systems is produced while the *H*∞ controller is suitable for linear time invariant (LTI) systems. Considering the actuator's dynamic together the robot's dynamic results in a linearized system as it follows.[2] Actuator used here is a DC motor with a dynamic equation as (2):

$$j_m\ddot{\theta}_m + b_m\dot{\theta}_m = \tau_m \tag{2}$$

Which is jm and bm are inertia and friction coefficient respectively. Actuator's shaft is connected to robot through a gearbox. So the angular velocity of motor is reduced with η as (3):

$$\tau = \eta\tau_m\,,\quad \dot{\theta} = \frac{1}{\eta}\dot{\theta}_m \tag{3}$$

Combining actuator and robot's together the equation (4) is obtained.

$$(M(\theta)+\eta^2 I_m)\ddot{\theta} + (V+\eta^2 b_m)\dot{\theta} + G = u \tag{4}$$

Because of nonlinear function of M, V, G equation (4) is nonlinear. But the more coefficient η2 the less nonlinearity properties. The model of robot and actuator can be considered as (5):

$$I_e\ddot{\theta} + b_e\dot{\theta} + G_{eff} = \tau \tag{5}$$

In which is Ie and be are effective inertia and effective friction's coefficient respectively and Geff represent the gravitational properties in the model.
For finding Ie,be and G the equation (5) is rewritten to a linear regression model as(6):

$$\begin{bmatrix}\ddot{\theta} & \dot{\theta} & 1\end{bmatrix}\begin{bmatrix} I_e \\ b_e \\ G_{eff}\end{bmatrix} = \begin{bmatrix}\tau\end{bmatrix} \tag{6}$$

IV. Modeling Of Uncertainty

The uncertainty contains all the unmodeled dynamics which is neglected in the identification process. Consider a set of transfer function for each joint named P0, if represent the nominal model, for yielding the nominal model and the uncertainty the concept of Multiplicative Perturbation is employed. Plant's dynamic is shown as (7):

$$\forall P(s) \in P, P(s) = (1 + \Delta(s)W(s))P_0(s) \tag{7}$$

Which is stable and fixed transfer function Ws is called Weighted Uncertainty function and stable variable transfer function satisfying (8) which is called Uncertainty [3].

$$\|\Delta\|_\infty \leq 1 \tag{8}$$

Using (8) and rewriting (7) as an inequality, equation (9) is obtained.

$$\left|\frac{P(j\omega)}{P(j\omega)} - 1\right| \leq |W(j\omega)|, \forall \omega \tag{9}$$

Plotting $\left|\frac{P(j\omega)}{P_0(j\omega)} - 1\right|$ for all transfer functions belongs to P, the nominal model and upper bound of inequality (9) as a Weighted Uncertainty are obtained. Here the process for joint (1) is displayed. For the other joints only the results are demonstrated (Tabel1).

Here the transfer functions identified for joint (1) are listed [1].

$$G_1 = \frac{1}{10.207S^2 + 261.71S}$$

$$G_3 = \frac{1}{12.662S^2 + 42.663S}$$

$$G_5 = \frac{1}{39.798S^2 + 58.643S}$$

$$G_7 = \frac{1}{18.392S^2 + 8.9693S}$$

$$G_9 = \frac{1}{13.212S^2 + 116.56S}$$

$$G_{11} = \frac{1}{10.547S^2 + 254.87S}$$

$$G_{13} = \frac{1}{115.32S^2 + 225.1S}$$

$$G_{15} = \frac{1}{3.0252S^2 + 131.02S}$$

$$G_2 = \frac{1}{12.95S^2 + 251.34S}$$

$$G_4 = \frac{1}{7.8903S^2 + 127.96S}$$

$$G_6 = \frac{1}{3.9107S^2 + 122.56S}$$

$$G_8 = \frac{1}{14.501S^2 + 86.767S}$$

$$G_{10} = \frac{1}{11.722S^2 + 170.7S}$$

$$G_{12} = \frac{1}{10.47S^2 + 391.11S}$$

$$G_{14} = \frac{1}{235.1S^2 + 1836.1S}$$

$$G_{16} = \frac{1}{8.0269S^2 + 11.876S}$$

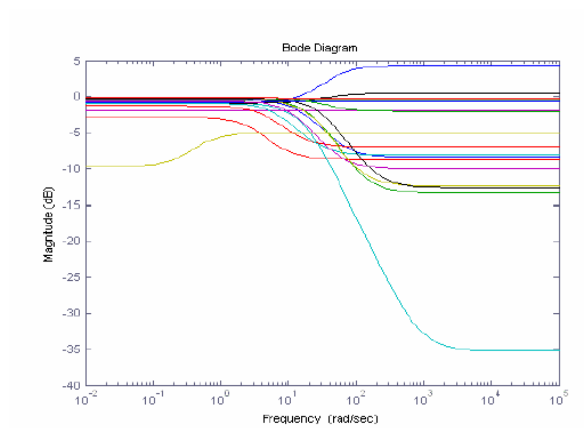

Fig1. Frequency Response of joint1

Here the nominal model and weighted uncertainty functions are displayed as (10), (11):

$$G_{16} = \frac{1}{8.0269S^2 + 11.876S} \tag{10}$$

$$w_2 = \frac{2.49(S+4)}{S+10} \tag{11}$$

V. Design Of $H\infty$ Controller

The purpose of this section is design of a $H\infty$ controller that is able to track a desired path. For this, the problem must be turned into synthesis problem using LFT theory (Figure 2) [3].

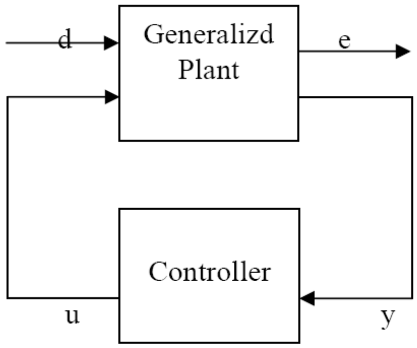

Fig2: Synthesis Problem

Using MATLAB software, hinfsyn command, $H\infty$ controller is achieved that is shown for Joint1 as (12):

$$k = \frac{-.00014709\,S^4 - .00054273\,S^3 + .011993\,S^2 + .028621\,S + .015021}{S^5 + 26.6344\,S^4 + 145.1021\,S^3 + 256.0195\,S^2 + 156.038\,S + 19.9415} \tag{12}$$

And the tracking performance of designed controller for joint(1) is as figure(3).

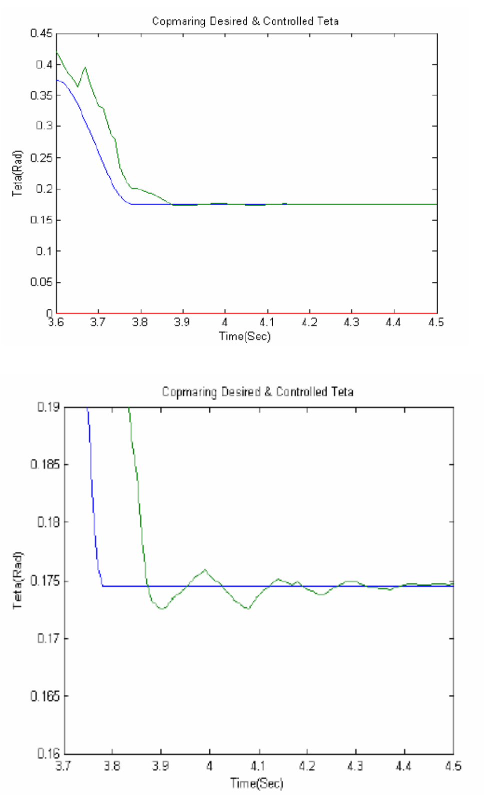

Fig3. Tracking of Joint1

Table1: Controller designed for robot's joints

Joint 2	$k = \dfrac{.092941S^4 + 1.6312S^3 + 3.9297S^2 + 2.8445S + .45305}{S^5 + 4.7779S^4 + 8.3866S^3 + 6.7826S^2 + 2.5475S + .37354}$
Joint 3	$k = \dfrac{.017109S^4 + .5294S^3 + 4.7619S^2 + 12.3006S + 5.1174}{S^5 + 10.2727S^4 + 33.6876S^3 + 40.3189S^2 + 19.0129S + 3.0269}$
Joint4	$k = \dfrac{.0055683S^4 + .29856S^3 + 4.6653S^2 + 24.8349S + 25.2891}{S^5 + 35.583S^4 + 142.7644S^3 + 205.9115S^2 + 112.6989S + 15.0324}$
Joint 5	$k = \dfrac{-.1357S^4 - 152.628S^3 + 791.9777S^2 + 2794.7674S + 1913.002}{S^5 + 1134.9885S^4 + 5756.2399S^3 + 10903.2091S^2 + 9432.5847S + 3174.4366}$
Joint 6	$k = \dfrac{.013983S^4 + .4295S^3 + 3.8253S^2 + 10.0948S + 8.1}{S^5 + 9.5814S^4 + 34.3793S^3 + 57.1285S^2 + 44.2315S + 13.6837}$
Joint 7	$k = \dfrac{.06476S^4 + 1.7633S^3 + 16.3645S^2 + 58.318S + 62.4129}{S^5 + 18.1138S^4 + 99.5352S^3 + 215.1741S^2 + 185.0612S + 54.588}$

Table2: Nominal model and Weighted uncertainty Function

	Nominal Model	Weighted Uncertainty
Joint 2	$plant = \dfrac{1}{90.553S^2 + 19.9276S}$	$w_2 = \dfrac{.97(2s+1)}{s+1}$
Joint 3	$Plant = \dfrac{1}{15.018S^2 + 51.666S}$	$w_2 = \dfrac{1.655(S+5)}{S+10}$
Joint 4	$Plant = \dfrac{1}{2.5637S^2 + 80.032S}$	$w_2 = \dfrac{5.9(S+1.68)}{S+10}$
Joint 5	$Plant = \dfrac{1}{.0025573S^2 + 2.8895S}$	$w_2 = \dfrac{1.09(S+1)}{S+1.1}$
Joint 6	$Plant = \dfrac{1}{2.4128S^2 + 4.8303S}$	$w_2 = \dfrac{3.31(S+3)}{S+10}$
Joint 7	$Plant = \dfrac{1}{.56752S^2 + .072421S}$	$w_2 = \dfrac{7.999(S+1)}{S+8}$

The nominal model and wheighted uncertainty functions related to joint 2 to joint 7 are shown in table 2.
As shown in figures(4-9) the error of the tracking for all of seven joints are in an acceptable region.

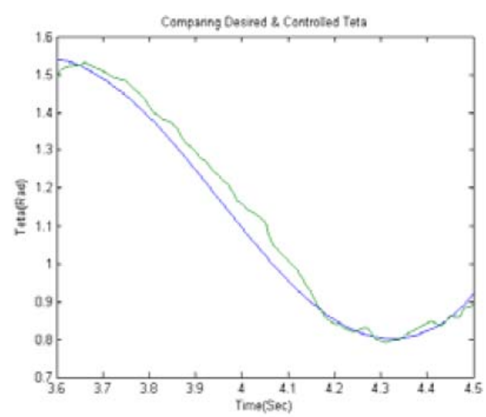

Fig4. Tracking of Joint 2

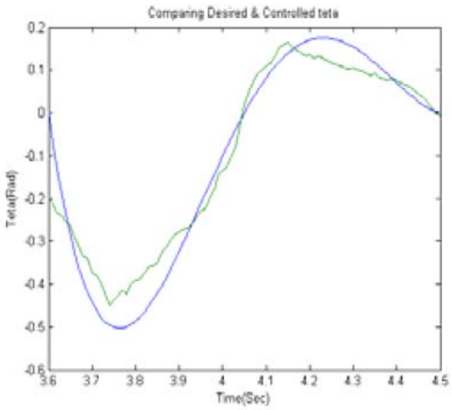

Fig5: Tracking of Joint 3

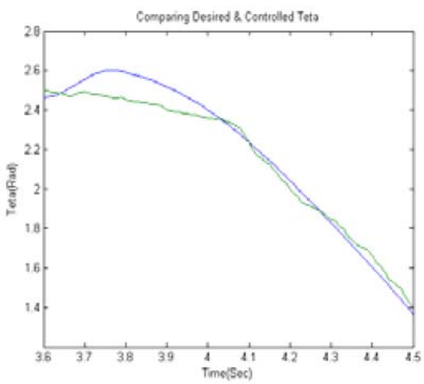

Fig6. Tracking of Joint 4

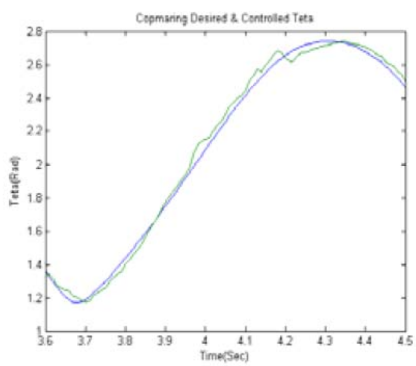

Fig7. Tracking of Joint5

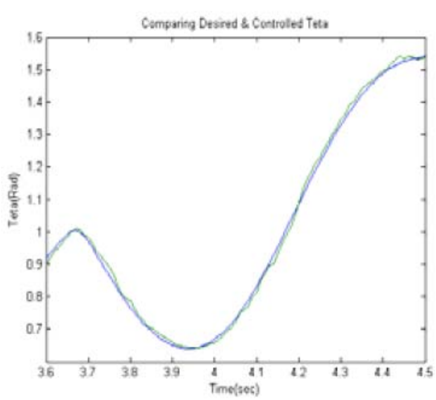

Fig8. Tracking of Joint 6

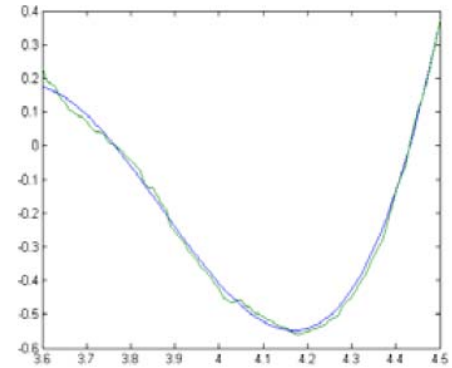

Figure9: Tracking of Joint 7

VI. CONCLUSIONS

In This paper, using linearization with actuator and also simulating the uncertainty as an unstructured, an $H\infty$ controller is designed for a 7 DOFs biped robot.

The results of the simulation for two robust stability and robust performance criteria, shown a good compromise with the mathematics solutions.

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