Modeling of Water Hammer Phenomenon in Simple Irrigation System and Comparing the Analytical to Experimental Results

Ahmad Bagheri
Majid Alitavoli
Mehdi Aghakashi
Salar Basiri
Amir Hajiloo

Mechanical Eng. Department
Guilan University, Rasht, Iran
E-mail: bagheri@guilan.ac.ir

Abstract

Any variation in fluid velocity in pipe line system, will produce compression waves in the system. These waves in result will produce higher pressure that the designed one inside the system which will propagate with the wave velocity. In this phenomenon which is called ' Water Hammer', factors such as opening and closing valves, starting or stopping the pump, stopping the turbine, increase or decrease in the amount of water, etc will cause water hammer.
Based on the above it is very important to model this phenomenon in order to firstly to determine the high pressure points to control water hammer. To this extent, a computer program is written. The goal of this program is to compare the mathematical and analytical modeling methods with the experimental results using a device made by Bergant.

Introduction

Research in the field of Water Hammer (WH) phenomenon has a century of background. The first mathematical model developed in 1925 by professor Rich who compared its result against experimental ones [1].
Streeter and Wylie were two pioneer researcher who spatially and completely done a survey about WH phenomenon in 1945 [2].
The importance of WH phenomenon steams from the fact that most fluid transferring systems in power stations, oil, and gas industries involves WH phenomenon.
The focus of this paper is to study the modeling of WH phenomenon in water supplying system and their accessories such as pumps, turbine, valves, and reservoirs. Also an attempt was made to model the effect of WH phenomenon in an experimental apparatus build by Bergant. Then the modeling results were compared against the experimental ones done by Bergant.

WH theory

As known, variation in fluid velocity ΔV produces variation in fluid pressure head $\Delta \mathrm{H}$. This variation in pressure will propagate by velocity a. In fact, as dedicated in Figure 1 the quantity of ΔV is negative, and therefore, the quantity of $\Delta \mathrm{H}$ is positive [1, 2].
The analysise is focused on a section of pipe shown as δL in Figure1. In this section δL is small and considered as desired size, but not smaller than the differential dL amount. Compressive wave and pipe deformation resulted by the variation in head of pressure $\Delta \mathrm{H}$ to be propagated by velocity a. Instead of considering the wave velocity in ratio to water velocity, it is considered in relevance to a fixed sight on the pipe. For relatively solid pipes, selection of any coordinate reference system will give similar results.

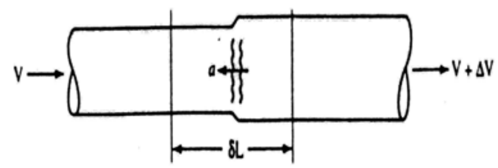


Figure1: The effect of WH in pipe

After writing the equation of linear momentum conservative principles for mass, we can calculate $\Delta \mathrm{H}$ in the pipe. This quantity is resulted from the following equations:

$$\Delta \mathrm{H} = -\frac{a}{g}\Delta V \quad (1)$$

In Which:

$$a = \frac{\sqrt{K/\rho}}{\sqrt{1+\frac{K}{E}\frac{D}{e}(C)}} \quad (2)$$

I this equation:
a = wave velocity (speed),
k = total elasticity coefficient of the medium,
e = wall thickness of pipe,
E = elasticity module of pipe,
ρ = Fluid density

Equations for entropy of mass and momentum to describe WH modeling

As noticed, for a sudden variation of ΔV in the pipe, we can calculate the variation in the head pressure $\Delta \mathrm{H}$. When expanded, we can calculate the head of the pressure and its velocity for each section of pipe in any time and under imposed loading conditions. To obtain this calculation we should use equations for mass and momentum entropy [2, 3].

$$\frac{\partial V}{\partial t}+\frac{1}{p}\frac{\partial p}{\partial s}+`g\frac{dz}{ds}+\frac{f}{2D}V|V|=0 \tag{3}$$

$$a^2\frac{\partial V}{\partial s}+\frac{1}{p}\frac{\partial p}{\partial t}=0 \tag{4}$$

In the above equations:
P = Fluid pressure
t = Time in sec
S = Pipe length
D= Internal diameter of the pipe
G = Gravity

Extraction of definite differential equation from basic differential equation for WH

The solving method for differential equations is a definite method. In definitive methods, two partial differential equations are subsumed by one normal differential equation. This deduction starts by substituting equations 3 and 4 by several linear equations of the same type. By using λ as the linear coefficient (Lagrange's coefficient), a combined linear equation is gained as follows:

$$\lambda\left(\frac{\partial V}{\partial t}+\frac{1}{P}\frac{\partial p}{\partial s}+g\frac{dz}{ds}+\frac{f}{2D}V|V|\right)+\left(a^2\frac{\partial V}{\partial s}+\frac{1}{P}\frac{\partial p}{\partial s}\right)=0 \tag{5}$$

When grouped, equation 5 will convert as follows:

$$\left(\lambda\frac{\partial V}{\partial t}+a^2\frac{\partial V}{\partial s}\right)+\left(\frac{1}{p}\frac{\partial P}{\partial t}+\frac{\lambda}{p}\frac{\partial P}{\partial s}\right)+\lambda g\frac{dz}{ds}+\frac{\lambda f}{2D}V|V|=0 \tag{6}$$

If $\lambda\frac{\partial V}{\partial t}+a^2\frac{\partial V}{\partial s}$ is substituted by $\lambda\frac{dV}{dt}$ then $\lambda\frac{ds}{dt}=a^2$, in addition if $\frac{1}{p}\frac{\partial p}{\partial t}+\frac{\lambda}{p}\frac{\partial p}{\partial s}$ is substituted by $\frac{1}{p}\frac{dp}{dt}$ then $\frac{\lambda}{p}=\frac{1}{p}\frac{ds}{dt}$.

From the above we obtain: $\lambda^2=a^2$ so $\lambda=\pm a$.
Now equation (6) can be rewritten once using $\lambda=+a$ and once using $\lambda=-a$, which loads us to a couple of the normal differential equations. By driving these equations on the velocity of wave and the variable $p=\gamma(H-z)$, we obtain:

$$\frac{dv}{dt}+\frac{g}{a}\frac{dH}{dt}+\frac{f}{2D}V|V|=0 \qquad \frac{ds}{dt}=+a \tag{7}$$

$$\frac{dv}{dt}-\frac{g}{a}\frac{dH}{dt}+\frac{f}{2D}V|V|=0 \qquad \frac{ds}{dt}=-a \tag{8}$$

To understand the solution process we refer to Figure 2.

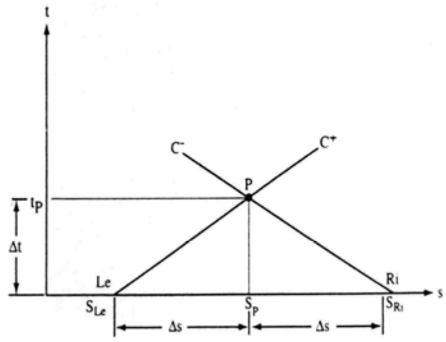


Figure2 : Extraction of definite differential equation procedure

Each point on the plane S-t which is shown as point P in an independent manner reserve to quantities like V and H. Then we draw the divertive lines C^+ and C^- at point P and continue them to intersect axis S on right and left side of point P. Notice that there exist two points over S axis which their horizontal distance from P is equal to Δs. These two points are shown as S_{Le} and S_{Ri}. Equation (6) is used in definite extent of C^+ and equation 7 is used in definite extent of C^-.

Extraction of definite algebraic equation from definite differential equation for WH

We write the equations (7) and (8) as limited difference type [2, 3].

$$C^- : (V_p-V_{Ri})-\frac{g}{a}(H_p-H_{Ri})+\frac{F\Delta t}{2D}V_{Ri}|V_{Ri}|=0 \tag{9}$$

$$C^+ : (V_p-V_{Le})+\frac{g}{a}(H_p-H_{Le})+\frac{F\Delta t}{2D}V_{Le}|V_{Le}|=0 \tag{10}$$

Also by writing the definite equations as the limited difference we have:

$$\Delta s=\pm a\Delta t \tag{11}$$

Now we solve the above equations by numerical limited difference method. First, we should decide to divide the pipe into several sections in the direction of S axis. If N is the member of division, then we have $\Delta s=\frac{L}{N}$. Based on the amount of Δs from equation 11 we can calculate Δt. Now we can make a network of definitive shown in Figure 3.

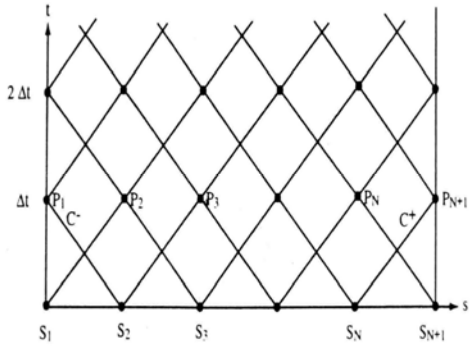


Figure 3: Complete definite equation solving

The points shown are selected to be in the direction of pipe with distance of Δs . The quantities V and H at these points are in primary conditions. Normally the primary conditions are part of V and H which is obtained from a stable flow condition at the start of a passing made. With the quantities Le and Fi we can solve equations 9 and 10 simultaneously. Then we can obtain quantities Hp and Vp from two points at period $t = \Delta t$. Boundary conditions at (S = 0) and (S = L) is used for calculating $H_{P_{N+1}}$, H_{PI} . Then all quantities for V and H in period $t = 2\Delta t$ by using the calculated quantities in the period $t = \Delta t$ as known quantities is earned. This process is used repeatedly to obtain V_{N+1}, H_{N+1}.

Analysis for the separation of relieved water and air columns

The separation of fluid column is occurred when the pressure of fluid drops below saturation pressure of steam.

The cross sign describes the separation phenomenon. Firstly we should prepare a model [7].

The simplest model for the separation, neglects the existence of unsaturated gases which may exit in pressures below the fluid pressure. Instead, it is assumed that fluid is continuously connected up to the steam pressure limit. After this point, bubble growth in constant pressure continues equally to steam pressure. Also this simple model requires assumptions about forming of bubble. Actually the process of forming bubble is very complex and hence the simulation is almost impossible. Therefore, we employ the simplest possible model. For this purpose, we assume that the bubble is depended to the pipe cross section and growth or death of the bubble is depended to the relational speed of bubble's wall. This survey (growth or death of bubble) need the analysis of boundary condition imposed. On the internal side of the bubble, the pressure is exactly equal to the steam flow. In each side in which separation occurs, two speeds are obtained. One speed is toward the up stream current of the bubble, and the other toward the down stream direction. Relation quantities and the direction of these speeds, determine the growth or death of bubble. This simple model is shown in Figure 4.

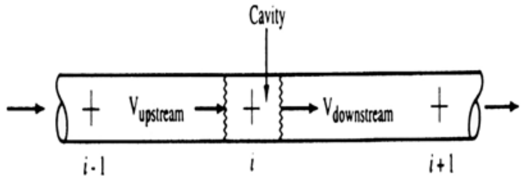


Figure 4 : Separation of relieved water and air model

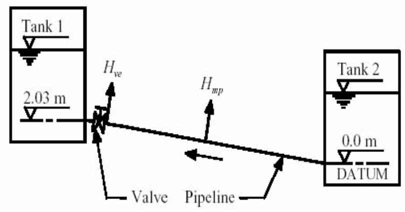
For analysis of this model the conservation of momentum is used. The collision occurred between two bubbles when moving by two different speeds. The results show an increase in the head due to the collision (Figure 4).

$$\Delta \mathrm{H} = \frac{g}{2g}\left(V_{Upstream} - V_{downstream}\right) \qquad (12)$$

This increase in head or $\Delta \mathrm{H}$, adds to each knot until the new pressure after bubble form up is determined.

Modeling WH phenomenon in Bergant test machine

The machine made by Bergant (Figure5) has two storage tanks, one pipe and a valve in the down side of the flow. Bergant tested WH phenomenon in different speeds of fluid and measured the pressure head in different points. Our intention was to compare Bergant's experimental results to analytical ones obtained for WH by definitive methods written by computer codes.



Experimental Apparatus (Pipe Length L = 37.2 m; Pipe Diameter D = 22 mm)

Figure 5: Bergant schematic

In this system, the pipe length was 37.2 m, diameter 22 mm, wave velocity of 1319 m/s and the pressure head in the storage tanks was kept constant at 22 m height and the speed in pipe was 0.2 m/s.

If the bottom valve in t = 0.009 is closed the WH phenomenon has been occurred in the system, as referred in references [4, 5, 6].

Results and suggestions

By studying the graphs from analytical modeling and comparing them to the Bergant results, and also results from other researches, it is observed that the accuracy and our prepared programs are very vital. As a result we can model any water network by using computer codes.
The results of the experimental jobs of Bergant have been shown in Figures 6 (for the middle of pipe) and 8 (for valves).
Figures 7 and 9 shown the comparison of results for simulation, regarded for the middle of pipe and valve respectively. An acceptable compromise between two sets of results has been observed.

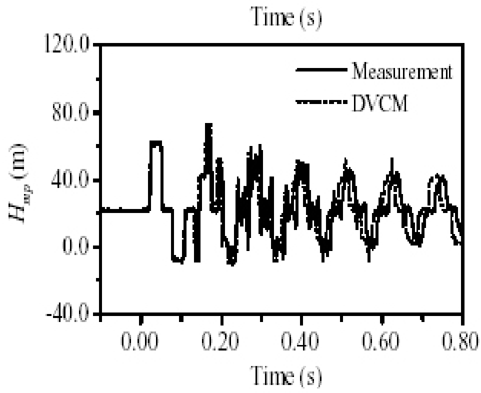


Figure 8 : Experimental Results of water hammer in valve

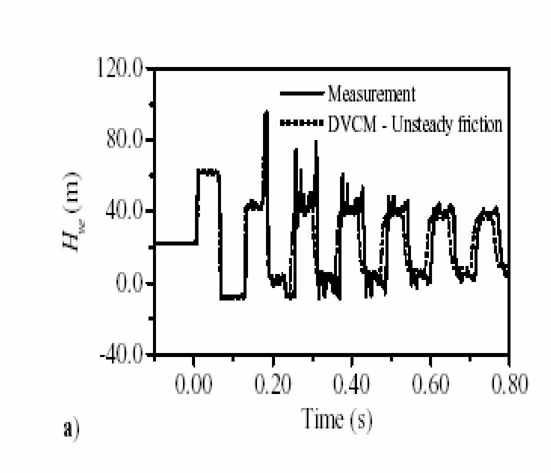


Figure 6: Experimental Results of water hammer in middle of pipe

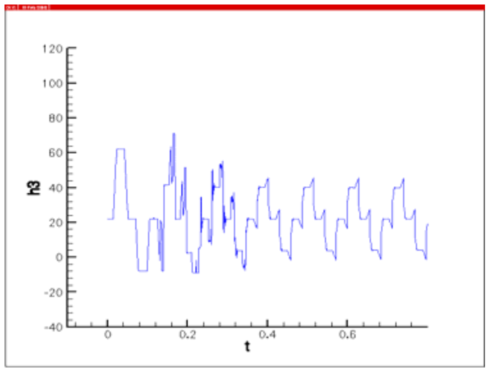


Figure 9: Simulated Results of water hammer in valve

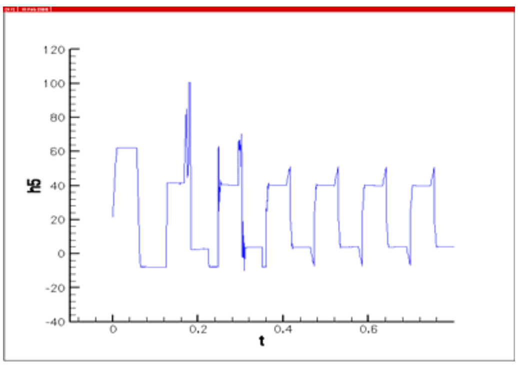


Figure 7: Simulated Results of water hammer in middle of pipe

References:

[1] Wylie, E.B., and Streeter, V.L. (1993). *Fluid transients in systems*. Prentice Hall, Englewood Cliffs, USA.

[2] Chaudhry, M.H. (1987). *Applied hydraulic transients.* Van Nostrand Reinhold Company, New York, USA.

[3] Wylie, E.B. (1984). Simulation of vaporous and gaseous cavitation. *Journal of Fluids Engineering*, ASME, 106(3), 307 - 311.

[4] Bergant A., and Simpson, A.R. (1999). Pipeline column separation flow regimes. *Journal of Hydraulic Engineering*, ASCE, 125(8), 835 - 848.

[5] Bergant, A., Simpson, A.R., and Vstkovsk ,عJ. (1999). Review of unsteady friction models in transient pipe flow. *Proceedings of the 9th International Meeting of the IAHR Work Group on the Behaviour of Hydraulic Machinery under Steady Oscillatory Conditions*, Brno, Czech Republic, Paper D1.

[6] Simpson, A.R., and Bergant, A. (1994). Numerical comparison of pipe-column-separation models. *Journal of Hydraulic Engineering*, ASCE, 120(3), 361 - 377.

[7] Aghakashi, M., (2005) The water hammer phenomenon modeling in simple piping systems, M.Sc. thesis, Guilan University, Iran.